Forecasting Applicant Demand and Optimising the Admission Plan of a Higher Education Institution Using Mathematical Modelling

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Abstract. *This paper addresses the problem of assessing applicants' interest in the study programmes of a higher education institution (HEI) and justifying its admission plan for the next academic year by means of a two-stage mathematical model. At the first stage, linear trend models are fitted by the least squares method to five years of application data, and the number of applications for 2027 is forecast; interest in a newly proposed programme is estimated from survey data. At the second stage, the allocation of admission places among programmes is formulated as a linear programming problem. The objective function is the expected number of employed graduates, and the constraints reflect faculty capacity, a minimum competition ratio, labour-market demand and the minimum group size. Calculations on illustrative data show that it is advisable to open an Artificial Intelligence programme, expand admission to Applied Mathematics and suspend separate admission to Statistics. Under the proposed plan, about 350 of the 400 admitted students are expected to find employment after graduation.*

Keywords: *higher education, admission planning, applicant demand, mathematical modelling, trend analysis, least squares method, linear programming, educational management.*

1. Introduction

The higher education system of the Republic of Uzbekistan is undergoing rapid reform: universities are being granted academic and financial autonomy, new study programmes are being opened, and admission quotas are increasingly set by the institutions themselves. Under these conditions, the admission plan becomes an important management decision for university leadership.

In practice, admission quotas are often set on the basis of the previous year's figures and expert judgement. This approach leads to two types of error:

- programmes with high demand receive too few places, and talented applicants move to other universities;
- programmes with declining demand see a sharp fall in competition, which lowers the quality of the student intake and the employment rate of graduates.

The quantitative assessment of applicant demand and the optimal allocation of admission places under limited resources are therefore topical problems.

The aim of the study is to develop a mathematical model for forecasting applicant demand for HEI study programmes and, on this basis, to justify the admission plan for the next academic year.

The object of the study is the admission process for bachelor's programmes at the Faculty of Mathematics of an HEI. **The subject of the study** is mathematical models for forecasting applicant demand and allocating admission places.

2. Literature Review

The regulatory framework for the development of higher education in Uzbekistan is set out in the Law "On Education" [1] and the Concept for the Development of the Higher Education System until 2030 [2]. The Concept sets the goals of increasing enrolment in higher education, aligning training with labour-market needs and expanding the autonomy of HEIs. These goals call for quantitative methods in admission planning.

Methods of time-series analysis and forecasting are covered in detail in the works of G. Box and G. Jenkins [3], D. Montgomery et al. [4], and R. Hyndman and G. Athanasopoulos [5]. These sources show that for short and stable series a linear trend model is a simple but reliable forecasting method.

Optimal resource allocation problems are based on the theory of linear programming, founded by G. Dantzig [6]; its applications are presented in the textbooks of F. Hillier and G. Lieberman [7] and H. Taha [8]. The use of operational research methods in education is reviewed by J. Johnes [9]. Stochastic models of student flows were developed in the work of D. Bartholomew [10].

Nevertheless, existing studies usually treat demand forecasting and the allocation of admission places as separate problems. In this paper they are combined in a single two-stage model in which the output of the first stage enters the constraints of the second stage directly.

3. Methodology

The study consists of two stages: forecasting the number of applications and allocating admission places optimally. The calculations were performed on illustrative data constructed to demonstrate the model; in practice, these data are replaced by the institution's actual admission statistics.

3.1. Initial data

The Faculty of Mathematics offers four bachelor's programmes. The numbers of applications received in 2022–2026 are given in Table 1.

Table 1. Number of applications by programme

Programme	2022	2023	2024	2025	2026
Mathematics	420	450	470	510	540
Applied Mathematics	600	720	830	950	1080
Mathematics Teaching Methodology	800	760	730	690	650
Statistics	150	160	155	165	170

The faculty is considering opening an Artificial Intelligence programme in 2027. Demand for it was estimated through a survey of 1,200 school leavers planning to apply to mathematics and IT programmes.

3.2. Stage 1: trend model

For each programme, the number of applications is expressed as a linear function of time:

$$y(t) = a + b\,t, \qquad t = 1,2,\dots,n$$

where y(t) is the number of applications in year t, b is the annual change in applications, and n = 5. The coefficients are found by the least squares method:

$$b = \frac{\sum_{i=1}^{n}(t_i - \bar{t})\,(y_i - \bar{y})}{\sum_{i=1}^{n}(t_i - \bar{t})^2}, \qquad a = \bar{y} - b\,\bar{t}$$

The goodness of fit is assessed with the coefficient of determination:

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2}$$

For the new programme, the expected number of applications is the share p of survey respondents who chose it multiplied by the number N of relevant applicants in the region:

$$\hat{y}_{new} = p \cdot N$$

3.3. Stage 2: linear programming model

The decision variables x_1, …, x_5 are the numbers of admission places allocated to the

programmes. The objective function is the expected number of employed graduates:

$$F = \sum_{i=1}^{5} w_i \, x_i \;\rightarrow\; \max$$

where w_i is the employment rate of graduates of programme i. The constraints are:

$$\sum_{i=1}^{5} x_i \le C \qquad \textit{(faculty capacity)}$$
$$x_i \le \hat{y}_i / k \qquad \textit{(competition ratio)}$$
$$x_i \le D_i \qquad \textit{(labour-market demand)}$$
$$x_i = 0 \;\vee\; x_i \ge m \qquad \textit{(minimum group size)}$$

Here C = 400 is the faculty's admission capacity, k = 3 is the minimum number of applicants per place, D_i is the demand for graduates according to employers and the ministry, and m = 25 is the minimum size of an academic group.

4. Results

4.1. Forecasting the number of applications

The calculation is illustrated for the Mathematics programme. Here $\bar{t} = 3$, $\bar{y} = 478$, $\Sigma(t_i - \bar{t})(y_i - \bar{y}) = 300$ and $\Sigma(t_i - \bar{t})^2 = 10$. Hence b = 30 and a = 478 − 30 · 3 = 388. The forecast for 2027 (t = 6) is:

$$y(6) = 388 + 30 \cdot 6 = 568$$

For the new programme, 15% of survey respondents chose Artificial Intelligence. With about 2,000 relevant applicants in the region, the expected number of applications is 0.15 · 2,000 = 300. The results for all programmes are summarised in Table 2.

Table 2. Trend models and forecast for 2027

Programme	Trend equation	R^2	Forecast for 2027, applications	Dynamics
Mathematics	y = 388 + 30t	0.991	568	growth, +30 per year
Applied Mathematics	y = 479 + 119t	0.999	1,193	rapid growth, +119 per year
Mathematics Teaching Methodology	y = 837 − 37t	0.998	615	decline, −37 per year
Statistics	y = 146.5 + 4.5t	0.810	174	unstable
Artificial Intelligence	survey	—	300	new programme

For three of the four programmes $R^2 > 0.99$, so the linear trend describes the data with high accuracy. For Statistics $R^2 = 0.81$, meaning that demand fluctuates from year to year.

Data from Tables 1 and 2 · 2022–2026 illustrative data, 2027 forecast

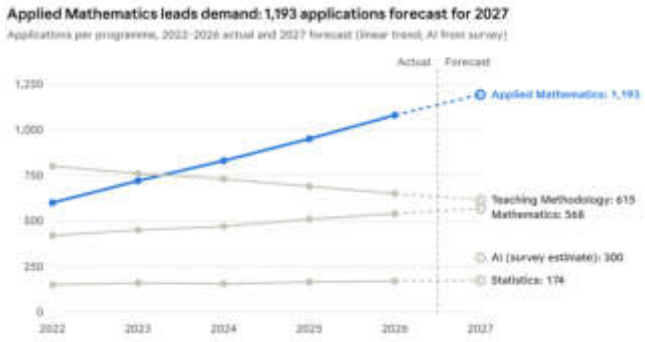


Figure 1. Dynamics of applications by programme and the forecast for 2027.

Demand for Applied Mathematics has grown 1.8-fold since 2022, while demand for Mathematics Teaching Methodology is steadily declining.

4.2. Optimal allocation of admission places

The model parameters are given in Table 3. The upper bound for each programme is the smaller of the competition and labour-market constraints.

The upper bounds sum to 440, which exceeds the faculty capacity (C = 400), so the capacity constraint is binding. The problem is a linear programme with a single joint constraint and variables bounded from above. Its optimal solution is obtained by filling the variables up to their upper bounds in descending order of w_i [7]. The resulting admission plan is given in Table 4.

The optimal value of the objective function is:

$$F_{\max} = 0.95 \cdot 60 + 0.92 \cdot 150 + 0.85 \cdot 120 + 0.75 \cdot 70 = 349.5$$

Table 3. Parameters of the optimisation model

Programme	Competition bound, $\hat{y}/3$	Labour-market demand, D_i	Upper bound	Employment rate, w_i
Mathematics	189	80	80	0.75
Applied Mathematics	397	150	150	0.92
Mathematics Teaching Methodology	205	120	120	0.85
Statistics	58	30	30	0.70
Artificial Intelligence	100	60	60	0.95

Table 4. Optimal admission plan for 2027

Programme	Decision	Admission places	Expected competition, applications per place
Artificial Intelligence	open new programme	60	5.0
Applied Mathematics	expand	150	8.0
Mathematics Teaching Methodology	keep	120	5.1
Mathematics	keep	70	8.1
Statistics	suspend admission	0	—
Total		**400**	

Thus, under the proposed plan, about 350 of the 400 admitted students are expected to find employment. In every programme that is opened, competition exceeds 3 applicants per place and intake exceeds the minimum group size of 25.

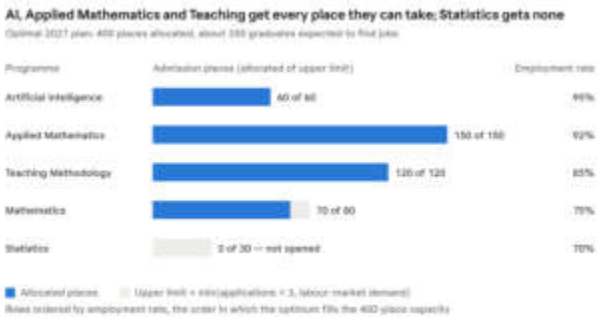


Figure 2. Optimal admission plan for 2027.

Data from Tables 3 and 4 · optimal solution of the linear programming problem

Three programmes are filled to their upper bounds; 10 places in Mathematics remain unallocated because of the capacity constraint.

5. Discussion

The results show that an admission plan can be built not only on the previous year's figures but also on demand dynamics and graduates' subsequent careers. The following conclusions follow from the model.

Marginal value of faculty capacity. In the optimal solution, only the Mathematics programme has not reached its upper bound. The dual price (shadow price) of the capacity constraint is therefore $w_1 = 0.75$: each additional place of faculty capacity increases the number of employed graduates by 0.75. This indicator can be used to justify decisions on expanding classroom space and teaching staff.

Robustness of the solution. The survey may have overstated interest in the new programme. If the actual number of applications is half the forecast (150), the competition bound falls to $x_5 \leq 50$. The 10 freed places move to Mathematics, and $F_{max} = 347.5$. The objective function changes by less than 1%, and the set of opened programmes stays the same. The decision to open the new programme is therefore robust to survey error.

Training of mathematics teachers. Demand for Mathematics Teaching Methodology is falling by an average of 37 applications per year. At the same time, the employment rate of its graduates is high (0.85), since schools continue to need mathematics teachers. To raise interest in this programme, it is recommended to increase the number of state-funded places and strengthen career guidance.

Limitations of the model. The proposed model relies on several simplifications:

- the linear trend is reliable only for short-term (1–2 year) forecasts; in the long term demand reaches saturation;
- stated intentions in a survey do not always turn into actual applications;
- tuition fees, the number of state grants, competing universities' offers and demographic changes are not included in the model;
- the calculations were performed on illustrative data, and the institution's actual admission statistics are required for practical use.

Future research may replace the trend model with ARIMA or multiple regression models [3, 5] and add financial indicators to the objective function, turning the problem into a multi-criteria optimisation problem.

6. Conclusion and Recommendations

This study developed a two-stage mathematical model for forecasting applicant demand for HEI programmes and optimising the admission plan. The model combines trend analysis and linear programming: the forecasts enter the constraints of the optimisation problem directly.

Based on calculations with illustrative data, the following recommendations were made:

1. Open an Artificial Intelligence programme with 60 places from 2027.
2. Expand admission to Applied Mathematics to 150 places and plan to increase faculty capacity in the coming years.
3. Keep 120 places in Mathematics Teaching Methodology and take measures to raise interest in it.
4. Suspend separate admission to Statistics and retain it as a specialisation within Applied Mathematics.

The proposed model is general and can be applied at the level of any faculty or institution. It is easy to implement in Microsoft Excel (with the Solver add-in) or in Python. This allows admission committees and academic affairs offices to update the admission plan on a scientific basis every year.

References

1. Law of the Republic of Uzbekistan "On Education". No. ZRU-637, 23 September 2020.
2. Decree of the President of the Republic of Uzbekistan "On approval of the Concept for the Development of the Higher Education System of the Republic of Uzbekistan until 2030". No. PF-5847, 8 October 2019.
3. Box G. E. P., Jenkins G. M., Reinsel G. C., Ljung G. M. Time Series Analysis: Forecasting and Control. 5th ed. Hoboken: Wiley, 2015.
4. Montgomery D. C., Jennings C. L., Kulahci M. Introduction to Time Series Analysis and Forecasting. 2nd ed. Hoboken: Wiley, 2015.
5. Hyndman R. J., Athanasopoulos G. Forecasting: Principles and Practice. 3rd ed. Melbourne: OTexts, 2021.
6. Dantzig G. B. Linear Programming and Extensions. Princeton: Princeton University Press, 1963.
7. Hillier F. S., Lieberman G. J. Introduction to Operations Research. 11th ed. New York: McGraw-Hill, 2021.
8. Taha H. A. Operations Research: An Introduction. 10th ed. Harlow: Pearson, 2017.
9. Johnes J. Operational Research in education. European Journal of Operational Research, 2015, 243(3), 683–696.
10. Bartholomew D. J. Stochastic Models for Social Processes. 3rd ed. Chichester: Wiley, 1982.