Modern approaches to teaching higher mathematics at technical higher education institutions: a mathematical model of the expected effectiveness of the flipped classroom method

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Abstract. *The paper reviews modern approaches to teaching higher mathematics at technical higher education institutions and selects the flipped classroom method for in-depth study. In order to forecast the expected effectiveness of the method quantitatively, a mathematical model of knowledge acquisition dynamics based on a first-order linear differential equation is developed. The model is parameterised by the assimilation, engagement and forgetting coefficients. Over one semester (15 weeks) the model predicts that the flipped classroom raises the mastery level from 56.8 % to 64.5 %, a gain of 7.6 percentage points (13.4 % in relative terms), corresponding to an effect size of d ≈ 0.51. The minimum sample size required to confirm the predicted effect in a pedagogical experiment is 61 students per group.*

Keywords: *higher mathematics, engineering education, flipped classroom, active learning, didactic modelling, differential equation, effect size, learning dynamics, statistical power.*

Introduction

Higher mathematics forms the fundamental core of engineering degree programmes: differential and integral calculus, differential equations, linear algebra and probability theory are prerequisites for mastering specialised subjects such as engineering mechanics, electrical engineering, materials science and control theory. Yet this subject consistently records some of the lowest pass rates, chiefly because of the abstraction of the material, the gap between the theoretical apparatus and engineering applications, heterogeneous school preparation and low learning motivation.

In the conventional lecture-and-seminar format, most of the contact time is spent transmitting information, while the cognitively most demanding stage — solving problems and reworking concepts — is shifted to independent study, where no instructional support is available. This allocation of the scarcest resource, expert attention, is didactically inefficient.

Over the past decade a number of modern approaches have emerged in response: problem-based and project-based learning, contextual instruction within the CDIO framework, visual work supported by computer algebra systems, adaptive and blended learning, just-in-time teaching, peer instruction, and the flipped classroom. The effectiveness of active learning in STEM disciplines is confirmed by large-scale syntheses of the evidence [2, 3], and

in Uzbekistan reform in this direction is linked to the objectives of the Concept for the Development of the Higher Education System until 2030 [1].

Most of the available literature, however, is descriptive or retrospectively empirical: results are recorded after a method has already been introduced. Large-scale pedagogical experiments are costly in both time and resources, which creates a need for predictive models that allow the expected effectiveness to be estimated quantitatively *before* the experiment is launched. It is this gap that motivates the present study.

The aim of the study is to construct a mathematical model of knowledge acquisition dynamics for the flipped classroom method, to use it to forecast the expected effectiveness of the method quantitatively, to validate the forecast against international meta-analytic evidence, and to determine the sample size required for empirical verification.

2. Materials and methods

2.1. The selected method. The flipped classroom was chosen as the object of the study [4, 5]. Its essence is the interchange of two stages of learning activity: theoretical material is acquired before class (through 10–15 minute video lectures, electronic lecture notes and automatically marked quizzes in the learning management system), while contact time is devoted entirely to problem solving, group discussion and professionally oriented tasks. In the topic "Applications of the definite integral", for instance, students acquire the definitions before class and in the classroom compute moments of inertia and hydrostatic pressure forces.

2.2. Formulation of the model. Let $y(t)$ denote the normalised mastery level of the competence associated with a teaching module, where $y \in [0; 1]$ and t is measured in semester weeks. The model rests on two assumptions: the rate of growth of knowledge is proportional both to the share of material not yet mastered, $(K - y)$, and to the student's cognitive engagement; and part of the knowledge acquired is lost through forgetting, proportionally to the amount currently held [9]. This yields a first-order linear differential equation:

$$dy/dt = \alpha u(K - y) - \beta y, \qquad y(0) = y_0, \tag{1}$$

where K is the didactic potential, that is, the maximum attainable level ($K \leq 1$); α is the assimilation coefficient; $u \in (0; 1]$ is the engagement coefficient, defined as the share of contact time during which the student is genuinely engaged in active cognitive work; and β is the forgetting coefficient.

For the initial condition $y_0 = 0$ the solution of equation (1) is

$$y(t) = (\alpha u K / \lambda)\cdot(1 - e^{-\lambda t}), \qquad \lambda = \alpha u + \beta, \tag{2}$$

and as $t \to \infty$ the system approaches the steady state

$$y_\infty = \alpha u K / (\alpha u + \beta). \tag{3}$$

Expression (3) yields the central didactic implication of the model: the limiting mastery level is determined not by the total volume of instructional time but by the *ratio between engagement and forgetting*. This is precisely the quantity on which the flipped classroom acts, since transferring contact time from passive listening to active problem solving raises u while leaving the nominal number of contact hours unchanged.

2.3. Parameter estimates. The following values were adopted: $K = 1$; $\alpha = 0.20$ week^{-1}; $\beta = 0.06$ week^{-1}, based on modern replications of the forgetting curve [9]; and $T = 15$ weeks. Drawing on time-on-task observations of classroom activity, the engagement coefficient

was set to $u_1 = 0.50$ for conventional instruction and $u_2 = 0.65$ for the flipped classroom.

2.4. Effectiveness criteria. The comparative effectiveness of the two methods was assessed through the absolute difference $\Delta = y_2(T) - y_1(T)$, the relative gain, and Cohen's effect size [10]:

$$d = [y_2(T) - y_1(T)] / \sigma, \quad (4)$$

where $\sigma = 0.15$ is the within-group standard deviation of the normalised mastery measure. The sample size required to confirm the effect at $\alpha = 0.05$ with a power of $1 - \beta = 0.80$ was obtained from

$$n = 2(z_{1-\alpha/2} + z_{1-\beta})^2 / d^2. \quad (5)$$

The analytical solution (2) was verified independently by fourth-order Runge–Kutta integration with step $h = 0.01$; the two results agreed to six decimal places, confirming the correctness of the computations.

3. Results

The values computed from (2) and (3) are presented in Table 1 and Figure 1. The limiting levels are $y_\infty = 62.5$ % for conventional instruction and $y_\infty = 68.4$ % for the flipped classroom.

Table 1. Modelled dynamics of the mastery level over one semester

Semester week, t	0	3	6	9	12	15
Conventional, $y_1(t)$, %	0.0	23.8	38.6	47.7	53.3	56.8
Flipped, $y_2(t)$, %	0.0	29.7	46.5	56.1	61.4	64.5
Difference Δ, pp	0.0	5.9	6.8	7.4	7.5	7.6
Relative gain, %	—	24.8	17.6	15.5	14.1	13.4

Note: computed from equation (2) with α = 0.20; β = 0.06; K = 1.

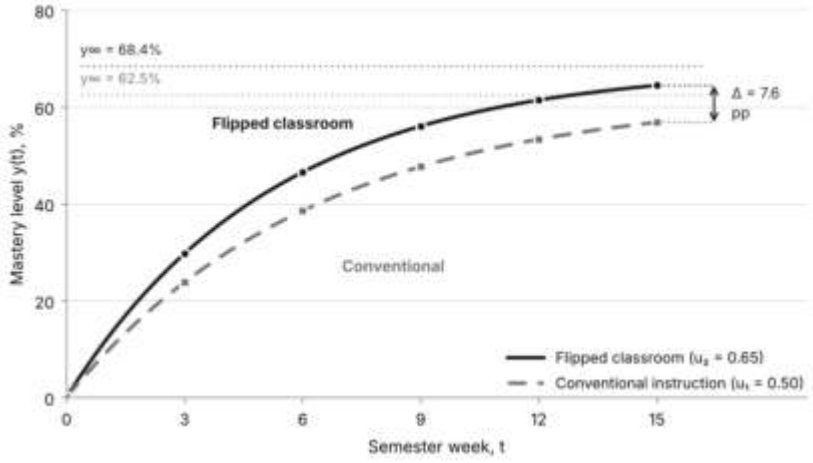


Figure 1. Modelled dynamics of the mastery level and its limiting values

At the end of the semester ($T = 15$ weeks) the mastery level reaches 56.8 % under conventional instruction and 64.5 % under the flipped classroom. The absolute difference is Δ = 7.6 percentage points and the relative gain is 13.4 %. From (4), the effect size is $d = 0.51$, which corresponds to a medium effect in Cohen's classification [10]. Applying (5), detecting this effect with adequate statistical power requires at least 61 students per group, or 122 in total.

A further implication concerns time rather than attainment: under the flipped classroom the

56.8 % level that conventional instruction reaches only at the end of the semester is attained already in week 9.3 — a saving of about 5.7 weeks, or 38 % of the course.

A sensitivity analysis was carried out using the partial derivative of (3) with respect to u, namely $\partial y_\infty/\partial u = \alpha\beta K/(\alpha u + \beta)^2$. It equals 0.469 at $u = 0.50$ and falls to 0.248 at $u = 0.80$, so the model reproduces a saturation effect: the first increments in engagement yield the largest returns. Table 2 summarises the scenario analysis across the plausible range of u_2.

Table 2. Scenario analysis of the predicted effect as a function of engagement

Engagement u_2	$y_2(T)$, %	Δ, pp	Effect size d	n per group
0.55	59.7	2.8	0.19	444
0.60	62.2	5.4	0.36	124
0.65 (base case)	64.5	7.6	0.51	61
0.70	66.5	9.7	0.65	38

Note: Δ and d are computed relative to the conventional baseline $y_1(T) = 56.8$ % ($u_1 = 0.50$).

4. Discussion

The forecast produced by the model ($d \approx 0.51$) falls within the range reported in published meta-analyses. Cheng, Ritzhaupt and Antonenko, pooling 115 comparisons, estimated an overall effect size of $g = 0.19$ [6]; Jang and Kim report $ES = 0.35$ for higher education [7]; and van Alten and co-authors obtained $g = 0.36$ [8]. Meta-analyses restricted to mathematics report higher values still.

What is particularly notable is that the sensitivity of the forecast to u_2 tracks the empirical evidence closely. As Table 2 shows, $u_2 = 0.55$ yields $d = 0.19$, $u_2 = 0.60$ yields $d = 0.36$, and $u_2 = 0.65$ yields $d = 0.51$. The estimates reported in [6] and in [7, 8] therefore correspond almost exactly to the low and intermediate scenarios of the model. This is evidence of external validity, and it also explains the high heterogeneity of the meta-analytic literature: studies carried out under the same label differ substantially in the engagement actually achieved in the classroom.

Jang and Kim found that mathematics and engineering benefit less from flipped instruction than other disciplines [7]. For higher mathematics it is therefore prudent to treat $u_2 = 0.65$ as an optimistic upper bound and $u_2 = 0.60$ as a conservative working estimate.

The model also yields a specific practical conclusion. If 30 % of students do not complete the pre-class stage, the group's effective engagement falls to $u_2 = 0.605$ and the effect size contracts from 0.51 to 0.37 — almost a third of the expected benefit. Mechanisms that enforce pre-class preparation, such as a compulsory entry quiz or rating points tied to it, are therefore not an optional refinement but a decisive condition of effectiveness.

Sensitivity to β is much lower: as the forgetting coefficient varies from 0.04 to 0.09 both curves fall together, the difference narrowing only from 7.8 to 7.3 percentage points and the effect size from 0.52 to 0.49. The predicted advantage is thus robust to uncertainty in this parameter.

Limitations. The model rests on several simplifications. u is treated as constant, whereas engagement may decline as the novelty effect wears off; the group is assumed homogeneous, so differences in prior preparation are not represented; only cognitive outcomes are addressed, and not the motivational and

interpersonal gains for which meta-analyses report larger effects [7]; and α, β and u were estimated from the literature and require calibration against institutional data.

5. Conclusions

1. Among modern approaches to teaching higher mathematics at technical universities, the flipped classroom was selected and modelled by a differential equation of knowledge formation linking the learning outcome to three interpretable parameters: the assimilation, engagement and forgetting coefficients.

2. Over one semester the method is expected to raise the mastery level from 56.8 % to 64.5 %, a gain of 7.6 percentage points or 13.4 % in relative terms, with an effect size of $d \approx 0.51$, which is a medium effect.

3. The temporal advantage is about 5.7 weeks: the end-of-semester level of conventional instruction is reached in week 9.3.

4. The predictions agree with international meta-analyses across the model's scenario range and explain their heterogeneity by differences in cognitive engagement.

5. Failure to complete pre-class preparation reduces the effect size from 0.51 to 0.37, so entry control must be treated as a mandatory element of implementation.

6. Empirical verification requires at least 61 students per group. Further work should calibrate the parameters on experimental data and extend the model to differences in prior preparation.

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