# Mathematical Modeling and Forecasting of the Annual Efficiency of Flat-Plate Solar Collectors in the Fergana Valley

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***Abstract.*** *A south-facing flat-plate solar collector in Fergana (40.39° N, 71.79° E) is predicted to deliver about 909 kWh/m² of useful heat per year at its optimal tilt of 30°, a mean annual efficiency of 51%. The forecast is built only from ground-station climate normals: monthly sunshine duration and air temperature. Global horizontal irradiation was estimated with the Ångström–Prescott equation and split into beam and diffuse parts with the Erbs monthly correlation. It was transposed to the tilted plane with the isotropic Liu–Jordan model and distributed over the day with the Collares-Pereira–Rabl ratios. Collector output was computed in steps of about 2 minutes for a mean day of each month with the Hottel–Whillier–Bliss equation and an incidence angle modifier. The estimated global horizontal irradiation is 1,614 kWh/m² per year. Monthly efficiency ranges from 21% in December to 63% in July and August, so the collector is a summer-dominated heat source. Raising the inlet temperature from 30 °C to 60 °C lowers annual efficiency from 60% to 36%. Each 5% loss of optical efficiency from dust lowers annual yield by about 6.6%. The method suits sites in the Fergana Valley that have sunshine records but no pyranometer.*

***Keywords:*** *flat-plate solar collector; thermal efficiency; Ångström–Prescott equation; sunshine duration; tilt angle; dust soiling; Fergana Valley; Uzbekistan*

## Introduction

Solar water heating is one of the cheapest ways to cut gas and electricity use for hot water in Uzbekistan. The Fergana Valley is densely populated and has a continental climate. Sunshine duration in the Fergana region is reported at 2,900–3,100 hours per year [1], while the long-term station normal for Fergana city is 2,648 hours [2]. Ground-adjusted satellite estimates for six Uzbek sites range from 1,622 to 1,810 kWh/m² of global horizontal irradiation per year; the only valley site, Pap in Namangan region, receives 1,687 kWh/m² [3].

This creates a design problem. A collector's datasheet efficiency is measured at high irradiance with a small temperature difference, typically 70–80%. Over a real year, low winter irradiance, cold air, high inlet temperatures and dust push the delivered efficiency far below that figure. Designers in the valley need a yearly forecast, not a datasheet point.

Ground pyranometer records are scarce. The Fergana actinometric station has measured total radiation since 1961 [4], but most district weather stations record only sunshine duration and temperature. Satellite databases help, but their deviation from ground sums in Uzbekistan grows outside the April–September season [5].

A method that uses only routinely observed variables is therefore useful.

The aim of this study is to forecast the monthly and annual thermal efficiency of a flat-plate collector in Fergana from ground climate normals alone. The specific objectives are: (i) to estimate monthly irradiation on horizontal and tilted planes; (ii) to find the tilt angle that maximises annual useful heat; and (iii) to quantify how inlet temperature, dust and the radiation-model coefficients change the forecast.

## Materials and Methods

### Site and input data

The site is Fergana city (40.39° N, 71.79° E, about 590 m a.s.l.), which has a cold desert climate (Köppen BWk) [2]. Two ground-based data sets were used; no satellite or reanalysis radiation product was used.

**Sunshine duration:** station normals for 1961–1990, from 95 h in December to 362 h in July, 2,648 h per year [2].

**Air temperature:** monthly mean daily minimum and maximum for 1991–2020 [18], from −2.5 / 4.9 °C in January to 20.8 / 35.2 °C in July.

Each month was represented by its recommended mean day [16]. Calculations were coded in Python (NumPy).

### Global horizontal irradiation

Monthly mean daily extraterrestrial irradiation on a horizontal surface was computed from the solar constant $G_{sc} = 1367$ W/m², the declination $\delta$, the latitude $\varphi$ and the sunset hour angle $\omega_s$ [12]:

$$H_0 = \frac{24 \cdot 3600\, G_{sc}}{\pi}\left(1 + \frac{0.033\cos(360\,n)}{365}\right)\left(\cos\varphi\cos\delta\sin\omega_s + \frac{\pi\,\omega_s}{180}\sin\varphi\sin\delta\right) \quad (1)$$

Global horizontal irradiation $H$ followed from the Ångström–Prescott equation [6, 7], where $S$ is mean daily sunshine duration and $S_0$ the astronomical day length:

$$\frac{H}{H_0} = a + b\,\frac{S}{S_0} \quad (2)$$

No locally calibrated coefficients were found for Fergana, so the FAO-56 general values $a = 0.25$ and $b = 0.50$ were used [8]. Their effect is tested in Section 3.4.

### Diffuse fraction and tilted-plane irradiation

The monthly diffuse fraction was obtained from the clearness index $K_T = H/H_0$ with the Erbs correlation for $\omega_s \le 81.4°$ [9]; the companion form was used for $\omega_s > 81.4°$:

$$\frac{H_d}{H} = 1.391 - 3.560\,K_T + 4.189\,K_T^2 - 2.137\,K_T^3 \quad (3)$$

Daily totals were distributed over the day with the Collares-Pereira–Rabl ratio $r_t$ for global and the Liu–Jordan ratio $r_d$ for diffuse irradiance [10, 11]. Irradiance on a south-facing plane of tilt $\beta$ was found with the isotropic sky model, with ground albedo $\rho_g = 0.2$:

$$G_T = G_b R_b + G_d\,\frac{1+\cos\beta}{2} + G\,\rho_g\,\frac{1-\cos\beta}{2} \quad (4)$$

### Collector model

Useful heat per unit area was calculated with the Hottel–Whillier–Bliss equation [12–14], with losses clipped at zero when the collector cannot gain heat:

$$q_u = \max\{0,\ F_R(\tau\alpha)_n\left[K_{\tau\alpha,b}\,G_b R_b + K_{\tau\alpha,d}\,(G_T - G_b R_b)\right](1-d) - F_R U_L\,(T_{in} - T_a)\} \quad (5)$$

The beam incidence angle modifier was $K_{\tau\alpha,b} = 1 - b_0\,(1/\cos\theta - 1)$ with $b_0 = 0.10$ [15]. Diffuse and ground-reflected radiation used the same expression at an effective angle of 60° ($K_{\tau\alpha,d} = 0.90$). The parameters represent a typical glazed collector with a selective absorber (Table 1).

**Table 1. Collector and operating parameters.**

| Parameter | Symbol | Value |
|---|---|---|
| Optical efficiency (normal incidence) | $F_R(\tau\alpha)_n$ | 0.75 |
| Heat loss coefficient | $F_R U_L$ | 4.5 W/(m²·K) |
| Incidence angle modifier coefficient | $b_0$ | 0.10 |

| | | |
|---|---|---|
| Inlet fluid temperature (base case) | $T_{in}$ | 40 °C |
| Dust loss of optical efficiency (base case) | $d$ | 0 |
| Orientation | — | due south |

Ambient temperature $T_a$ followed a cosine between the monthly mean minimum and maximum, peaking at 15:00 solar time. Equations were integrated from sunrise to sunset in steps of about 2 minutes.

### Performance indicators

Monthly and annual efficiency were defined as delivered heat over incident tilted-plane irradiation, where $N_{days,m}$ is the number of days in month $m$:

$$\eta_{year} = \frac{\sum_{m=1}^{12} N_{days,m}\ Q_{u,m}}{\sum_{m=1}^{12} N_{days,m}\ H_{T,m}} \qquad (6)$$

The optimal tilt was the angle, scanned from 0° to 70° in 1° steps, that maximised annual useful heat.

## Results

### Monthly irradiation and efficiency

At the optimal 30° tilt the collector receives 1,780 kWh/m² per year and delivers 909 kWh/m², an annual efficiency of 51.1% (Table 2). The model gives 1,614 kWh/m² per year on the horizontal plane. The clearness index rises from 0.42 in December to 0.66 in August, which reflects cloudy, humid winters and clear summers.

**Table 2. Monthly results for a south-facing collector at 30° tilt, $T_{in}$ = 40 °C, clean glazing.**

| Month | $T_a$ (°C) | $K_T$ | $H$ (kWh/m²) | $H_T$ (kWh/m²) | $Q_u$ (kWh/m²) | $\eta$ (%) |
|---|---|---|---|---|---|---|
| Jan | 1.2 | 0.43 | 55.5 | 82.4 | 18.5 | 22.5 |
| Feb | 3.8 | 0.44 | 68.8 | 90.1 | 26.6 | 29.6 |
| Mar | 10.5 | 0.46 | 108.1 | 124.6 | 50.6 | 40.6 |
| Apr | 16.7 | 0.51 | 147.1 | 154.0 | 77.5 | 50.3 |
| May | 21.6 | 0.56 | 192.1 | 186.7 | 104.7 | 56.1 |
| Jun | 26.1 | 0.63 | 218.8 | 204.7 | 124.7 | 60.9 |
| Jul | 28.0 | 0.65 | 227.9 | 216.8 | 136.5 | 63.0 |
| Aug | 26.6 | 0.66 | 207.3 | 212.6 | 133.2 | 62.7 |
| Sep | 21.8 | 0.65 | 161.0 | 186.1 | 110.2 | 59.2 |
| Oct | 15.0 | 0.57 | 110.1 | 144.5 | 72.5 | 50.2 |
| Nov | 8.0 | 0.51 | 68.4 | 103.3 | 38.9 | 37.6 |
| Dec | 2.7 | 0.42 | 48.6 | 74.3 | 15.6 | 21.0 |
| Year | 15.2 | — | 1,613.6 | 1,779.9 | 909.4 | 51.1 |

The six warm months (April–September) supply 687 kWh/m², or 76% of the annual heat. December and January together supply only 34 kWh/m². In those months losses to cold air consume most of the absorbed energy, and efficiency falls to about one third of its summer value.

### Optimal tilt angle

Annual useful heat peaks at a tilt of 30°, about 10° below the latitude (Figure 1). A horizontal collector delivers 782 kWh/m², or 14% less. The optimum is flat: any tilt between 20° and 40° stays within 1.7% of the maximum. Seasonal use shifts the optimum to 18° for April–September and to 52° for October–March.

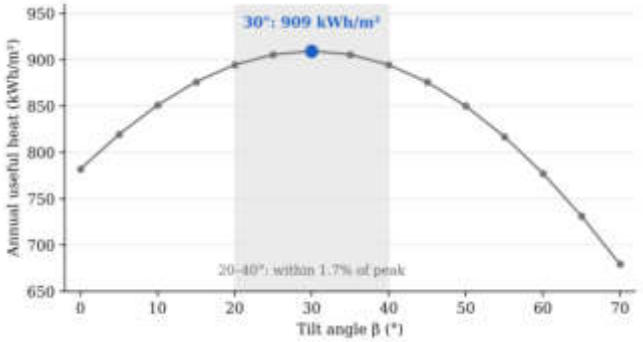



**Figure 1. Annual useful heat versus tilt angle ($T_{in}$ = 40 °C, clean glazing).**

### Inlet temperature and dust

Inlet temperature and dust are the two operating factors with the largest effect (Table 3). Each 10 K rise in inlet temperature removes about 140–160 kWh/m² per year. Each 5% loss of optical efficiency to dust removes about 59 kWh/m², or 6.6% of the clean-glazing yield.

**Table 3. Sensitivity of annual results at 30° tilt.**

| Case | $Q_u$ (kWh/m²·yr) | $\eta_{year}$ (%) | Change in $Q_u$ vs. base |
|---|---|---|---|
| Base: $T_{in}$ = 40 °C, clean | 909.4 | 51.1 | — |
| $T_{in}$ = 30 °C | 1,067.7 | 60.0 | +17.4% |
| $T_{in}$ = 50 °C | 766.9 | 43.1 | −15.7% |
| $T_{in}$ = 60 °C | 637.9 | 35.8 | −29.9% |
| Dust loss $d$ = 5% | 849.6 | 47.7 | −6.6% |
| Dust loss $d$ = 10% | 790.1 | 44.4 | −13.1% |
| Dust loss $d$ = 15% | 730.9 | 41.1 | −19.6% |
| Ångström $a$ = 0.20, $b$ = 0.55 | 861.4 | 50.5 | −5.3% |
| Ångström $a$ = 0.30, $b$ = 0.45 | 958.6 | 51.7 | +5.4% |

### Sensitivity to Ångström coefficients

Changing the coefficients by ±0.05 shifts horizontal irradiation by ±3.3% (1,561–1,666 kWh/m²) and useful heat by about ±5.4%. Annual efficiency changes by less than 0.7 percentage points. Efficiency is therefore a more robust output of the model than the absolute heat yield.

## Discussion

The modelled horizontal irradiation of 1,614 kWh/m² is plausible for the valley. It is 4.3% below the ground-adjusted estimate of 1,687 ± 34 kWh/m² for Pap, the nearest assessed site, about 80 km away in the same valley [3]. Differences in site, data period and method can easily explain a gap of this size. A ground pyranometer comparison is still needed before the absolute values can be called validated.

The optimal tilt of 30° is about 10° below the latitude, whereas latitude tilt is the usual rule of thumb. The reason is the strong seasonal asymmetry of the Fergana climate. In December–February the sun shines only about a third of the possible hours, so a steep, winter-oriented tilt gains little, while clear summers reward a flatter plane. The flat optimum means that roof pitch, not the exact tilt, can usually decide installation.

The gap between datasheet and annual efficiency is the main practical finding. The modelled collector has an optical efficiency of 75%, yet its annual efficiency is 51% at 40 °C inlet and only 36% at 60 °C. For domestic hot water systems the inlet temperature is set by tank stratification and user demand. Low-temperature preheating, a stratified tank and a separate auxiliary heater all raise the delivered efficiency.

Dust is a local concern because of agriculture, unpaved roads and occasional dust haze in the valley. Field studies elsewhere report optical losses of several percent to tens of percent between cleanings, depending on climate and tilt [17]. In this model, a 10% loss removes 13% of annual heat. Periodic cleaning in the dry summer months, when most energy is collected, therefore has the largest payoff.

**Limitations.** (i) The Ångström coefficients are the FAO-56 general values, not calibrated for Fergana; Section 3.4 shows a ±5% effect on heat yield. (ii) Sunshine (1961–1990) and temperature (1991–2020) normals come from different periods, and the long-term variation of radiation at Fergana [4] is not modelled. (iii) The isotropic sky model tends to underestimate irradiance on tilted planes. (iv) Collector parameters are typical rather than taken from a tested product, and a constant inlet temperature replaces a full system simulation with tank and load. (v) Mean-day calculation does not capture day-to-day variability, which the utilizability method or hourly typical-year data would address.

## Conclusions

The performance of a flat-plate solar collector in the Fergana region can be forecast using only sunshine duration and ambient air temperature, without requiring satellite-derived data. The estimated annual global horizontal irradiation is 1,614 kWh/m². A south-facing collector installed at the optimal tilt angle of 30° receives approximately 1,780 kWh/m² of annual solar irradiation and produces 909 kWh/m² of useful thermal energy, achieving an annual efficiency of 51% at an inlet temperature of 40 °C.

The collector's efficiency varies considerably throughout the year, ranging from 21% in December to 63% in July. The period from April to September accounts for approximately 76% of the annual useful heat production, demonstrating the significant influence of seasonal climatic conditions on solar thermal performance.

The analysis indicates that collector tilt angles between 20° and 40° result in an annual energy yield reduction of less than 1.7% compared with

the optimal configuration. However, seasonal optimization produces different results, with optimal tilt angles of 18° for summer applications and 52° for winter applications. Furthermore, increasing the collector inlet temperature by every 10 K reduces annual useful heat production by approximately 16–17%, while a 5% reduction in optical performance due to dust accumulation decreases annual energy yield by approximately 6.6%.

Future research should focus on calibrating the Ångström coefficients using actinometric measurements recorded in the Fergana region and experimentally validating the proposed model using an actual flat-plate solar collector.

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